Int. J. Heat Mass Transfer. Vol. 19, pp. 125–126. Pergamon Press 1976. Printed in Great Britain

LETTERS TO THE EDITORS

COMMENTS TO "THE ONSET OF NATURAL CONVECTION FROM TIME-DEPENDENT PROFILES"

(*Received* 6 *February* 1975)

NOMENCLATURE

- g, gravitational acceleration;
- K, function of n or S defined by equation (7);
- l, time-dependent characteristic length;
- n, exponent;
- $Q(t)$, time-dependent heat generation rate;
- $q_s(t)$, heat flux at the heated surface;
- Ra_l, time-dependent Rayleigh number;
- S, shape factor defined by equation (3);
- s, Laplace transform variable of time;
- t, elapsed time;
- z, vertical distance from the heated surface.

Greek symbols

- α, thermal diffusivity;
- β, coefficient of thermal expansion;
- $\Gamma(n)$, gamma function;
- $\theta(t, z)$, rise in fluid temperature above bulk temperature;
- $\bar{\theta}(s, z)$, Laplace transform of $\theta(t, z)$ with respect to t;
- $\theta_s(t)$, rise in surface temperature above bulk temperature;
- $\bar{\theta}_s(s)$, Laplace transform of $\theta_s(t)$;
- μ, viscosity;
- ξ, variable;
- ρ, density;
- τ, shape variable defined by equation (4);
- τ_Q, reduced time based on heat generation rate.

DAVENPORT and King [1] experimentally found that Rayleigh numbers at which the onset of convection was detected in the fluid heated from below were scarcely dependent on the behavior of the temperature of the heated surface if the characteristic length l and Rayleigh number Ra_l were defined as follows:

$$l = 2\int_0^\infty \theta(t, z)\,dz/\theta_s(t) \tag{1}$$

$$Ra_l = \rho g \beta \theta_s(t) l^3/\mu\alpha. \tag{2*}$$

This fact is very interesting. In order to identify the behavior of the surface temperature, they also introduced the shape factor

$$S = \tau/t, \tag{3}$$

where

$$\tau = \int_0^t \theta_s(\xi)\,d\xi/\theta_s(t). \tag{4*}$$

If $\theta_s(t) \propto t^n$,

$$l = K\sqrt{(\alpha t)} \tag{5}$$

$$S = 1/(n+1), \tag{6}$$

where

$$K = 2\Gamma(n+1)/\Gamma(n+3/2) = 2\Gamma(1/S)/\Gamma(1/S+1/2). \tag{7}$$

They will probably have determined l through equations (3), (4), (5) and (7).

*There is a notational error in equation (4) of [1] and some error in either equation (5) of [1] or its explanation.

The shape factor S is independent of time and consequently appears to be convenient so long as $\theta_s(t)$ is assumed to be proportional to t^n. But it may become time-dependent for other types of transient of the surface temperature. We would point out that it is the variable τ rather than S that is of direct importance in correlating the influence of unsteady temperature profile on the onset of convection, since as shown later the unsteady temperature profile in the fluid is considerably well approximated in terms of τ prior to the onset of convection, namely, one value of τ corresponds to one temperature profile independent of the behavior of the surface temperature. The shape factor S will not be any more than a dimensionless expression of the shape variable τ. In fact, equation (5) can be written as

$$l \simeq 2\sqrt{(\alpha\tau)}, \tag{8}$$

since

$$K \simeq 2\sqrt{S}. \tag{9}$$

Sakurai, Mizukami and Shiotsu [2] introduced the reduced time

$$\tau_Q = \int_0^t Q(\xi)\,d\xi/Q(t) \tag{10}$$

in the course correlating the experimental data of transient burnout. Sakurai and Mizukami [3, 4] experimentally found that heat-transfer coefficient by transient conduction can be expressed approximately in terms of the reduced time, and further tried to describe in terms of it the temperature variation in a heating material and surrounding solid induced by a transient of heat input.

We restrict our argument to cases in which a variation of the surface temperature is prescribed. The Laplace transform of the conduction solution is obtained as

$$\bar{\theta}(s, z) = \bar{\theta}_s(s)\exp[-\sqrt{(s/\alpha z)}]. \tag{11}$$

Using the same procedure as proposed in [4], an approximate temperature profile in the fluid is obtained as

$$\theta(t, z)/\theta_s(t) = \exp[-z/\sqrt{(\alpha\tau)}]. \tag{12}$$

The approximate temperature profile is compared in Fig. 1 with the strict one for $\theta_s(t) \propto t^n$. The temperature profile near the heated surface will be more precisely expressed if

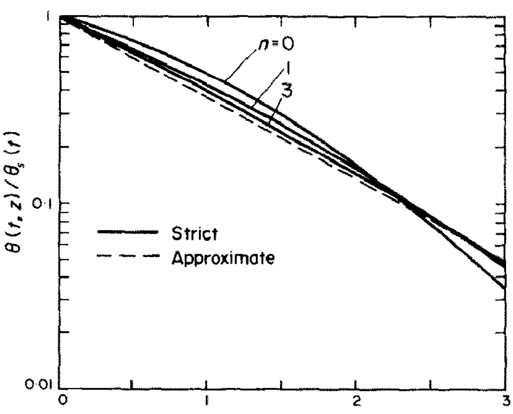

FIG. 1. Approximate and strict temperature profiles for $\theta_s(t) \propto t^n$.

the shape variable τ is defined by

$$\tau = \int_0^t q_s(\xi)\,d\xi/q_s(t). \qquad (13)$$

Equation (12) will be available for many practical cases. Then the time-dependent characteristic length l will necessarily be nearly proportional to $\sqrt{(\alpha\tau)}$ even if it is defined in other ways instead of equation (1). The characteristic length defined by equation (1) is identical, in an approximate sense, with such the thermal boundary-layer thickness as $\theta(t, l)/\theta_s(t) = 1/e^2$.

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Int. J. Heat Mass Transfer. Vol. 19, p. 126. Pergamon Press 1976. Printed in Great Britain

CORRELATIONS FOR HEAT TRANSFER TO VARIABLE PROPERTY FLUIDS IN TURBULENT PIPE FLOW

(*Received* 18 *June* 1975)

THE PAPER by Sleicher and Rouse [1] presents the most recent of many surveys of heat-transfer correlations for variable-property fluids in smooth pipes. After a review of previous correlations, and careful selection of the most reliable experimental data for comparison, the following equation was proposed:

$$Nu_b = 5 + 0{\cdot}015 Re_f^a Pr_w^b \quad \text{for} \quad \begin{matrix} 0{\cdot}1 < Pr < 10^5 \\ 10^4 < Re < 10^6 \end{matrix}$$

where

$$a = 0{\cdot}88 - 0{\cdot}24/(4 + Pr_w)$$

and

$$b = 1/3 + 0{\cdot}5\,e^{-0{\cdot}6 Pr_w}.$$

Our first query concerns the meaning of the suffix in Re_f. Does the suffix apply to ρ and μ or only to μ? This is always a dilemma. There is a natural reluctance to base Re upon a produce $\rho_f V$ which is not equal to the average mass velocity; on the other hand, to retain the average mass velocity is to introduce the unwarranted assumption that density variations across the pipe have no effect.

Consider the application of the above correlation to a fluid for which the thermal conductivity increases with temperature, the other properties being constant. As the wall-to-bulk temperature difference is increased, for a fixed bulk temperature, k_w increases but according to the correlation, the heat-transfer coefficient decreases. To illustrate this characteristic of the correlation: at $Pr_b = 1{\cdot}0$, if a thermal conductivity variation was introduced such that $k_w/k_b = 10$, the calculated heat-transfer coefficient would *decrease* by a factor 6·3. It is physically unreasonable that the injection of a higher conductivity into the wall region should reduce the heat-transfer coefficient.

The answer may come back that the correlation is a reasonable fit to the best experimental data, which do not include a fluid for which the conductivity variation predominates. In fitting the experimental data perhaps the anomolous dependence of the correlation on thermal conductivity variation is compensated by an over-allowance for the variation of some other property. This indeed is our principal point: that such a correlation can be regarded only as a summary of a particular set of experimental data, and not, despite the generality of its appearance, as possessing any fundamental basis that would enable it to be used with confidence for predictive purposes outside the conditions of the experiments on which it has been based.

From first principles one may write

$$Nu_b = F\left(Re_b,\ Pr_b,\ \frac{\rho_w}{\rho_b},\ \frac{\mu_w}{\mu_b},\ \frac{C_{pw}}{C_{pb}},\ \frac{k_w}{k_b}\right)$$

recognising also that the form of the variation of the properties with temperature is significant. Even for fluids whose properties vary with temperature in a monotomic and "regular" manner (i.e. excluding near-critical fluids in particular) it is certain that any realisation of such a correlation which must include interactions between the different physical property ratios, would be forbiddingly complicated. Fundamental objections to some of the simpler forms of correlation which have been tried for many years have been discussed in [2] and we do not believe that these simpler forms can be regarded as other than convenient summaries of particular sets of data.

As a more positive proposal we believe it would be a useful step forward to shed some light on the influence of variations of the four physical properties, separately and in combination. We are presently engaged upon this task.

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